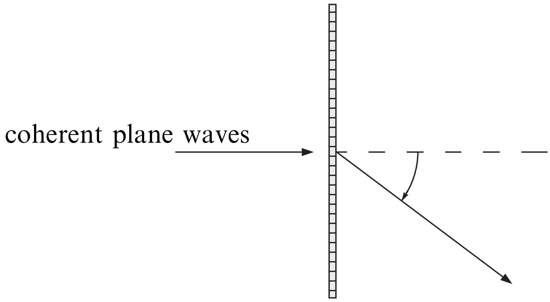
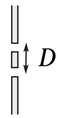
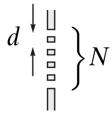
8.3 Fraunhofer diffraction

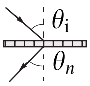
Gratings[a]

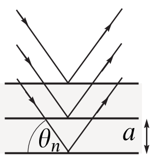


Young's double slits[b]	$I(s)=I_0\cos^2\dfrac{kDs}{2}$	(8.24)	$I(s)$ diffracted intensity; I_0 peak intensity; θ diffraction angle; s $=\sin\theta$; D slit separation; λ wavelength
N equally spaced narrow slits	$I(s)=I_0\left[\dfrac{\sin(Nkds/2)}{N\sin(kds/2)}\right]^2$	(8.25)	N number of slits; k wavenumber $(=2\pi/\lambda)$; d slit spacing
Infinite grating	$I(s)=I_0\displaystyle\sum_{n=-\infty}^{\infty}\delta\left(s-\frac{n\lambda}{d}\right)$	(8.26)	n diffraction order; δ Dirac delta function
Normal incidence	$\sin\theta_n=\dfrac{n\lambda}{d}$	(8.27)	θ_n angle of diffracted maximum
Oblique incidence	$\sin\theta_n+\sin\theta_{\rm i}=\dfrac{n\lambda}{d}$	(8.28)	$\theta_{\rm i}$ angle of incident illumination
Reflection grating	$\sin\theta_n-\sin\theta_{\rm i}=\dfrac{n\lambda}{d}$	(8.29)	
Chromatic resolving power	$\dfrac{\lambda}{\delta\lambda}=Nn$	(8.30)	$\delta\lambda$ diffraction peak width
Grating dispersion	$\dfrac{\partial\theta}{\partial\lambda}=\dfrac{n}{d\cos\theta}$	(8.31)	
Bragg's law[c]	$2a\sin\theta_n=n\lambda$	(8.32)	a atomic plane spacing



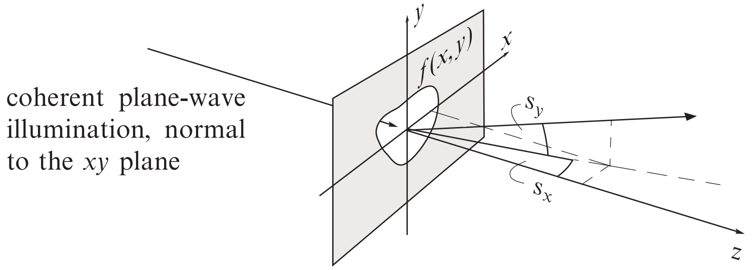






[a] Unless stated otherwise, the illumination is normal to the grating.
[b] Two narrow slits separated by D.
[c] The condition is for Bragg reflection, with $\theta_n=\theta_{\rm i}$.

Aperture diffraction



General 1-D aperture[a]	$\psi(s) \propto \int_{-\infty}^{\infty} f(x)\mathrm{e}^{-\mathrm{i}ksx}\,\mathrm{d}x$	(8.33)	ψ diffracted wavefunction I diffracted intensity θ diffraction angle s $=\sin\theta$
	$I(s) \propto \psi\psi^*(s)$	(8.34)	
General 2-D aperture in (x,y) plane (small angles)	$\psi(s_x,s_y) \propto \iint_{\infty} f(x,y)\mathrm{e}^{-\mathrm{i}k(s_x x+s_y y)}\,\mathrm{d}x\,\mathrm{d}y$	(8.35)	f aperture amplitude transmission function x,y distance across aperture k wavenumber ($=2\pi/\lambda$) s_x deflection $\parallel$ xz plane s_y deflection $\perp$ xz plane
Broad 1-D slit[b]	$I(s) = I_0 \dfrac{\sin^2(kas/2)}{(kas/2)^2}$	(8.36)	I_0 peak intensity a slit width (in x) λ wavelength
	$\equiv I_0 \,\mathrm{sinc}^2(as/\lambda)$	(8.37)	
Sidelobe intensity	$\dfrac{I_n}{I_0} = \left(\dfrac{2}{\pi}\right)^2 \dfrac{1}{(2n+1)^2} \qquad (n>0)$	(8.38)	I_n nth sidelobe intensity
Rectangular aperture (small angles)	$I(s_x,s_y) = I_0 \,\mathrm{sinc}^2 \dfrac{as_x}{\lambda} \,\mathrm{sinc}^2 \dfrac{bs_y}{\lambda}$	(8.39)	a aperture width in x b aperture width in y
Circular aperture[c]	$I(s) = I_0 \left[\dfrac{2J_1(kDs/2)}{kDs/2}\right]^2$	(8.40)	J_1 first-order Bessel function D aperture diameter
First minimum[d]	$s = 1.22\dfrac{\lambda}{D}$	(8.41)	λ wavelength
First subsid. maximum	$s = 1.64\dfrac{\lambda}{D}$	(8.42)	
Weak 1-D phase object	$f(x) = \exp[\mathbf{i}\phi(x)] \simeq 1 + \mathbf{i}\phi(x)$	(8.43)	$\phi(x)$ phase distribution $\mathbf{i}$ $\mathbf{i}^2 = -1$
Fraunhofer limit[e]	$L \gg \dfrac{(\Delta x)^2}{\lambda}$	(8.44)	L distance of aperture from observation point Δx aperture size

[a] The Fraunhofer integral.
[b] Note that $\mathrm{sinc}\,x = (\sin\pi x)/(\pi x)$.
[c] The central maximum is known as the "Airy disk."
[d] The "Rayleigh resolution criterion" states that two point sources of equal intensity can just be resolved with diffraction-limited optics if separated in angle by $1.22\lambda/D$.
[e] Plane-wave illumination.